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***Abstract*—We consider the downlink (DL) of a satellite communication system with single-antenna mobile receivers and static users that is modeled as a vector broadcast channel (BC). While for static users perfect channel state information (CSI) is available, only the rank-one covariance matrices of the channels to mobile users are known. In this scenario, we design beamforming techniques that are able to deal with full CSI users and statistical CSI users at the same time. In this paper, we employ zero-forcing (ZF) techniques with regard to the mobile users' channels. Under this assumption, a duality w.r.t. the signal-to-interference-and-noise-ratio (SINR) between the vector BC and an appropriately constructed vector multiple-access channel (MAC) is established. Based on the observation that an interference function can be defined in the dual vector MAC that is standard, an iterative solution for the quality-of-service (QoS) power minimization and the rate balancing problem is found.**

***Index Terms*—Vector BC; complete and statistical CSI users; BC-MAC duality; QoS power minimization; rate balancing**

I. Introduction

In this work, we consider the DL of a satellite mobile communication system, where a satellite equipped with a large number of antenna elements conveys independent information to single-antenna receivers on the earth's surface. Here, we abandon the fixed beamforming of state-of-the-art satellite systems [1]. Instead, we model the DL as a vector BC according to [2] and focus on advanced physical layer interference-mitigation techniques for an adaptive beamformer design.

Contrary to standard BC models with perfect channel knowledge at the transmitter, we have to deal with two groups of receivers in satellite mobile communications (cf. [2]). While for the *static users* perfect CSI is a valid assumption, only the statistics of channels to moving *mobile users* is available due to the large round-trip time (see Section II). In this BC setup, we aim at the *stochastically robust* design of beamforming techniques which are able to deal with static and mobile users at the same time [3]. Considering that satellite communication systems are strictly limited with regard to the available transmit power, a robust QoS optimization and a balancing problem are formulated based on achievable and ergodic rate requirements.

Unfortunately, when incorporating ergodic rate constraints, we cannot resort to the efficient fixed-point methods that are known for the purely complete CSI vector BC (e.g., [4], [5], and [6]) due to the lack of a general BC to MAC rate duality for statistical transmitter CSI [7]. However, with a sufficiently large number of antennas at the transmitter and rank-one channel covariance matrices for the mobile users (see Section II), we can force the transmitter to cause no interference at the mobiles, e.g., by introducing ZF constraints w.r.t. the mobile receivers' channels. Incorporating these ZF constraints into the effective transmit strategies, an SINR duality [8] between the considered vector BC and an appropriately constructed vector MAC is established. Based on the observations that an interference function can be defined in the dual vector MAC that is standard (cf. [5] and [6]), an iterative solution for the QoS power minimization with the mentioned ZF constraints is found. This result also enables an algorithmic solution of the corresponding balancing problem formulation.

The remainder of this work is structured as follows. In Section II, the satellite mobile vector BC is introduced. The robust QoS power minimization, the balancing problem, and its partial ZF reformulations are presented in Section IV. Feasibility conditions for the QoS problem are given in Section V. In Section VI, the duality between the considered BC and an appropriately constructed MAC is established that leads to the standard interference formulation in Section VII. In Section VIII, the implemented algorithmic solutions for solving the QoS and balancing optimization are presented and some numerical results are finally given in Section IX.

II. System and Channel Model

In the considered satellite vector BC, an N antenna transmitter, i.e., the satellite, conveys independent information to K single antenna receivers, viz. mobile and static users. The data signal $s_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ for user k is linearly precoded with the beamformer $\boldsymbol{t}_k \in \mathbb{C}^N$. The resulting transmitted signal $\boldsymbol{x} = \sum_{k=1}^K \boldsymbol{t}_k s_k$ propagates over the vector channel $\boldsymbol{h}_k^{\mathrm{H}} \in \mathbb{C}^{1\times N}$ and suffers from additive Gaussian noise $n_k \sim \mathcal{N}_{\mathbb{C}}(0,\sigma_k^2)$. Therefore, the received signal of user k reads as

$$y_k = \boldsymbol{h}_k^{\mathrm{H}} \sum\nolimits_{i=1}^K \boldsymbol{t}_i s_i + n_k.$$

We do not make the standard assumption that the channel state $\boldsymbol{h}_k^{\mathrm{H}}$ of all receivers is available error-free at the transmitter. While perfect CSI is a valid assumption for the static users $k \in \mathcal{S} \subset \{1,\ldots,K\}$, e.g., fixed ground stations with proper reflectors that have line of sight to the satellite, the statistics of the channel state to the moving mobile receivers $k \in \mathcal{M} = \{1,\ldots,K\} \setminus \mathcal{S}$ has to be estimated

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at the receiver and fed back to the transmitter. Here, we consider that the mobiles reside in an urban environment and suffer from heavy shadowing, scattering, and diffraction from buildings, trees, and moving vehicles close to these users [9]. Therefore, we assume that the transmitter is only aware of the covariance matrix $\boldsymbol{C}_{\boldsymbol{h}_k} = \mathrm{E}[\boldsymbol{h}_k \boldsymbol{h}_k^{\mathrm{H}}]$ of the zero-mean channel $\boldsymbol{h}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{C}_{\boldsymbol{h}_k})$ for $k \in \mathcal{M}$.

Fortunately, as a specialty in satellite communications, the channel covariance matrices of these statistical CSI users are rank-one. The spatial channel characteristics to mobiles remains essentially constant because of the large distance from the satellite to the earth's surface, whereas the effective gain varies dependent on the current scattering and shadowing environment of the mobile user when the satellite works at Ka-band [9]. Hence, the kth channel covariance matrix is completely characterized by the scaled dyadic product of its dominant eigenvector $\boldsymbol{v}_k$, i.e., $\boldsymbol{C}_{\boldsymbol{h}_k} = \psi_k^2 \boldsymbol{v}_k \boldsymbol{v}_k^{\mathrm{H}}$.

III. Achievable and Ergodic Rates

We aim at a power efficient beamformer design in terms of achievable rates. For a given set of beamforming vectors $\{\boldsymbol{t}_k\}_{k=1}^K$, the rates are given by the mutual information

$$R_k \triangleq I_k(s_k; y_k) = \log_2(1 + \mathrm{SINR}_k^{\mathrm{BC}}) \tag{1}$$

and with the standard definition of the SINR in the BC

$$\mathrm{SINR}_k^{\mathrm{BC}} = \frac{|\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k|^2}{\sigma_k^2 + \sum_{i \neq k} |\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_i|^2}. \tag{2}$$

Note that we implicitly assume in (1) that the transmitter and the receiver have accurate knowledge about the channel states $\boldsymbol{h}_k$. Obviously, this assumption is invalid for the mobile users $k \in \mathcal{M}$, where we are only aware of the channels' statistics. Consequently, we cannot use (1) for these users. Instead, we resort to the *ergodic* mutual information which appropriately describes the mobiles' average rate. The kth user's ergodic mutual information $\bar{R}_k \triangleq \mathrm{E}_{\boldsymbol{h}_k}[I_k(s_k; y_k)]$ reads as (cf. [10])

$$\begin{aligned}\bar{R}_k = \tfrac{1}{\log(2)} \varsigma\Big(1 \Big/ \textstyle\sum_{i=1}^K \frac{\psi_k^2}{\sigma_k^2} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2\Big) \\ - \tfrac{1}{\log(2)} \varsigma\Big(1 \Big/ \textstyle\sum_{i \neq k} \frac{\psi_k^2}{\sigma_k^2} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2\Big),\end{aligned} \tag{3}$$

where we defined $\varsigma(x) \triangleq \mathrm{e}^x \, \mathrm{E}_1(x)$ and $\mathrm{E}_1(x) = \int_x^\infty \frac{\mathrm{e}^{-t}}{t} \mathrm{d}t$ is the exponential integral [11].

IV. Problem Formulation

For the beamformer design, we consider two robust problem formulations that are well established for error-free CSI at the transmitter and the receiver. If given QoS requirements, expressed as minimum (ergodic) rates, shall be fulfilled using minimum total transmit power, we end up with

$$\min_{\{\boldsymbol{t}_1, \ldots, \boldsymbol{t}_K\}} \sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \quad \mathrm{s.\,t.:} \begin{array}{ll} R_k \geq \rho_k & \forall k \in \mathcal{S}, \\ \bar{R}_k \geq \rho_k & \forall k \in \mathcal{M}. \end{array} \tag{4}$$

A formulation that is closely related to the QoS problem (4) in the perfect CSI case [12] is the balancing optimization

$$\max_{\beta, \{\boldsymbol{t}_1, \ldots, \boldsymbol{t}_K\}} \beta \quad \mathrm{s.\,t.:} \sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \leq P_{\mathrm{tx}}, \begin{array}{ll} R_k \geq \beta\rho_k & \forall k \in \mathcal{S}, \\ \bar{R}_k \geq \beta\rho_k & \forall k \in \mathcal{M}. \end{array} \tag{5}$$

Note that (5) always has a solution contrary to (4) which might be infeasible for $K > N$ and too large targets (cf. [13]).

In contrast to the perfect CSI case, above optimization problems—incorporating ergodic rate requirements—are difficult to solve in general. There exists no bijective map which connects the ergodic mutual information (3) with some SINR-like term. Thus, we can neither reformulate (4) into a power minimization with minimum SINR requirements, nor write (5) as an SINR balancing problem. Therefore, we cannot rely on the efficient optimization techniques of [6], [14].

To overcome this difficulty, we propose a partial ZF method under the assumption that $|\mathcal{M}| < N$. That is, we introduce additional ZF conditions w.r.t. the mobile users' channels:

$$\boldsymbol{v}_j^{\mathrm{H}} \boldsymbol{t}_k = 0 \quad \forall j \in \mathcal{M} \setminus \{k\}, \quad \forall k \in \{1, \ldots, K\}. \tag{6}$$

Satisfying these ZF constraints, we cause no interference at the mobile receivers. However, the beamformers that correspond to mobile users can cause interference at the static receivers.

Via a proper parameterization of the beamformers, (6) can be incorporated into (4) and (5). To this end, we define the sub-unitary bases $\{\boldsymbol{W}_k\}_{k=1}^K$, with $\boldsymbol{W}_k = \boldsymbol{W}$ for $k \in \mathcal{S}$, $\boldsymbol{W}^{\mathrm{H}} \boldsymbol{W} = \boldsymbol{I}_{N-|\mathcal{M}|}$, and $\boldsymbol{W}_k^{\mathrm{H}} \boldsymbol{W}_k = \boldsymbol{I}_{N-|\mathcal{M}|+1}$ for $k \in \mathcal{M}$, such that for all beamforming vectors $k \in \{1, \ldots, K\}$

$$\boldsymbol{t}_k = \boldsymbol{W}_k \boldsymbol{\tau}_k, \quad \boldsymbol{W}_k^{\mathrm{H}} \boldsymbol{v}_j = \boldsymbol{0} \quad \forall j \in \mathcal{M} \setminus \{k\}, \tag{7}$$

where $\boldsymbol{\tau}_k$ is the parameter vector intended for user k. Noting that the transmit power is $\sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 = \sum_{k=1}^K \|\boldsymbol{\tau}_k\|_2^2$, (4) and (5) with (6) can be recast in terms of $\{\boldsymbol{\tau}_k\}_{k=1}^K$.

The QoS power minimization (4) becomes

$$\min_{\{\boldsymbol{\tau}_1, \ldots, \boldsymbol{\tau}_K\}} \sum_{k=1}^K \|\boldsymbol{\tau}_k\|_2^2 \quad \mathrm{s.\,t.:} \begin{array}{ll} R_k \geq \rho_k & \forall k \in \mathcal{S}, \\ \bar{R}_k \geq \rho_k & \forall k \in \mathcal{M}. \end{array} \tag{8}$$

We remark that, by incorporating (7), the ergodic mutual information in (3) can be expressed as

$$\bar{R}_k^{\mathrm{BC}} = \tfrac{1}{\log(2)} \varsigma\big(1/\mathrm{SINR}_k^{\mathrm{BC}}\big), \ \mathrm{SINR}_k^{\mathrm{BC}} = \tfrac{\psi_k^2}{\sigma_k^2} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{W}_k \boldsymbol{\tau}_k|^2, \tag{9}$$

i.e., it is a bijective function of the 'SINR' defined in (9). Hence, (8) can equivalently be formulated as a power minimization with SINR requirements

$$\gamma_k(\rho_k) = \begin{cases} 2^{\rho_k} - 1 & k \in \mathcal{S} \\ 1/\varsigma^{-1}\big(\log(2)\rho_k\big) & k \in \mathcal{M} \end{cases} \tag{10}$$

and is solved (if feasible) via the usual methods. That is, we are able to specify the feasibility test of [13] for (8). Given feasibility, we construct a dual MAC for the considered BC with partial ZF and find the optimal solution of (8) with the *interference function framework* [15], [5] in the dual MAC.

The balancing optimization (5) can be rewritten as

$$\max_{\beta, \{\boldsymbol{\tau}_1, \ldots, \boldsymbol{\tau}_K\}} \beta \quad \mathrm{s.\,t.:} \sum_{k=1}^K \|\boldsymbol{\tau}_k\|_2^2 \leq P_{\mathrm{tx}}, \begin{array}{ll} R_k \geq \beta\rho_k & \forall k \in \mathcal{S}, \\ \bar{R}_k \geq \beta\rho_k & \forall k \in \mathcal{M}. \end{array} \tag{11}$$

Rewriting the rate targets of (9) as SINR requirements with $\gamma_k(\beta\rho_k)$, one can easily verify that (8) and (11) are inverse problems [15]. Let the optimum of (11) be denoted as

$\beta_{\text{opt}}(P_{\text{tx}})$ and the optimum of (8) as $P(\rho_1, \ldots, \rho_K)$. Then, $P(\beta_{\text{opt}}(P_{\text{tx}})\rho_1, \ldots, \beta_{\text{opt}}(P_{\text{tx}})\rho_K) = P_{\text{tx}}$ holds. Since $\beta_{\text{opt}}(P_{\text{tx}})$ is monotonically increasing in P_{tx}, (11) can be solved via a bisection where β is adjusted such that $P(\beta\rho_1, \ldots, \beta\rho_K)$ is equal to P_{tx} (see Section VIII).

Note that, by solving (8) and (11), we obtain conservative bounds for the actual power minimization and balancing problem formulation in (4) and (5), respectively.

V. Feasibility of the QoS Power Minimization

Note that (8) might not have a solution for given targets $\{\rho_k\}_{k=1}^K$ when $K > N$; even if partial ZF is applicable, i.e., $|\mathcal{M}| < N$ and $\{\boldsymbol{v}_k\}_{k\in\mathcal{M}}$ are linearly independent. However, following [13], i.e., expressing the rate targets as *minimum-mean-square-error* (MMSE) requirements [cf. (10)]

$$\varepsilon_k = \tfrac{1}{1+\gamma_k(\rho_k)} \quad \forall k \in \{1, \ldots, K\},$$

we can state a simple feasibility test for $\{\rho_k\}_{k=1}^K$. For this test, the channel signatures in $\mathcal{H} = \{\boldsymbol{h}_k\}_{k\in\mathcal{S}} \cup \{\boldsymbol{v}_k\}_{k\in\mathcal{M}}$ have to be regular: the elements in $\mathcal{H}$ are regular iff for any subset $\mathcal{K} \subseteq \mathcal{H}$, with $|\mathcal{K}| \leq N$, the vectors in $\mathcal{K}$ are linearly independent.

Theorem 1. *Assuming regular channels and above ZF conditions to be satisfied, the MMSE targets $\{\varepsilon_k\}_{k=1}^K$ are feasible iff they lie within a polytope with box constraints $0 \leq \varepsilon_k \leq 1\ \forall k$ and the sum constraint $\sum_{k=1}^K \varepsilon_k \geq \max\big(0, |\mathcal{S}| - (N - |\mathcal{M}|)\big)$ (cf. [13, Theorem 1]).*

The proof of this theorem directly follows the proof in [13] using the dual MAC formulation presented next, that incorporates the ZF conditions. In the remainder of this paper, we assume that the given rate targets $\{\rho_k\}_{k=1}^K$ are feasible.

VI. SINR Duality for Partial Zero-Forcing

Let the channels of the dual vector MAC be defined as $\boldsymbol{b}_k = \frac{1}{\sigma_k}\boldsymbol{h}_k$ for $k \in \mathcal{S}$ and $\boldsymbol{b}_k = \frac{\psi_k}{\sigma_k}\boldsymbol{v}_k$ for $k \in \mathcal{M}$. With the noise $\boldsymbol{\eta} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$, the received signal in the vector MAC is

$$\boldsymbol{r} = \sum_{k\in\mathcal{S}} \boldsymbol{b}_k\sqrt{p_k}\xi_k + \sum_{k\in\mathcal{M}} \boldsymbol{b}_k\sqrt{p_k}\xi_k + \boldsymbol{\eta} \in \mathbb{C}^N.$$

Here, $\xi_k \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ denotes the data signal for user k and $p_k \in \mathbb{R}_+$ is the power of the kth user in the MAC. To find a connection to the original vector BC with partial ZF, the receiver applies the equalizer $\boldsymbol{f}_k = \boldsymbol{W}_k\boldsymbol{\phi}_k \in \mathbb{C}^N$ to get the data signal estimate $\hat{\xi}_k = \boldsymbol{f}_k^{\mathrm{H}}\boldsymbol{r}$. Due to (7), the filters are orthogonal to the interfering channels of the mobile users in the MAC and the SINR reads as

$$\mathrm{SINR}_k^{\mathrm{MAC}} = \frac{|\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_k|^2 p_k}{\|\boldsymbol{\phi}_k\|_2^2 + \sum_{i\in\mathcal{S}\setminus\{k\}} |\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_i|^2 p_i}. \tag{12}$$

The respective rates are $R_k^{\mathrm{MAC}} = \log_2(1 + \mathrm{SINR}_k^{\mathrm{MAC}})$ for $k \in \mathcal{S}$ and $\bar{R}_k^{\mathrm{MAC}} = \frac{1}{\log(2)}\varsigma(1/\mathrm{SINR}_k^{\mathrm{MAC}})$ for $k \in \mathcal{M}$. In the following, we will show that there exists a one-to-one relationship between $\mathrm{SINR}_k^{\mathrm{BC}}$ and $\mathrm{SINR}_k^{\mathrm{MAC}}$.

Assume that some SINRs are achievable in the dual vector MAC. If we set

$$\boldsymbol{\tau}_k = \alpha_k\boldsymbol{\phi}_k \qquad \forall k \in \{1, \ldots, K\}, \tag{13}$$

we get from $\mathrm{SINR}_k^{\mathrm{BC}} = \mathrm{SINR}_k^{\mathrm{MAC}}$ that [cf. (2) and (12)]

$$\Big(\|\boldsymbol{\phi}_k\|_2^2 + \sum_{i\in\mathcal{S}\setminus\{k\}} |\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_i|^2 p_i\Big)\alpha_k^2 = p_k + \sum_{i\neq k} |\boldsymbol{b}_k^{\mathrm{H}}\boldsymbol{W}_i\boldsymbol{\phi}_i|^2 p_k\alpha_i^2$$

for $k \in \mathcal{S}$ and similar for $k \in \mathcal{M}$ that [cf. (9) and (12)]

$$\Big(\|\boldsymbol{\phi}_k\|_2^2 + \sum_{i\in\mathcal{S}} |\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_i|^2 p_i\Big)\alpha_k^2 = p_k.$$

The K scalar equations can be combined to

$$\boldsymbol{\Phi}\boldsymbol{a} = \boldsymbol{p}, \tag{14}$$

with $\boldsymbol{a} = [\alpha_1^2, \ldots, \alpha_K^2]^{\mathrm{T}}$, $\boldsymbol{p} = [p_1, \ldots, p_K]^{\mathrm{T}}$, and $\boldsymbol{a}, \boldsymbol{p} \in \mathbb{R}_+^K$. Since

$$[\boldsymbol{\Phi}]_{k,\ell} = \begin{cases} \|\boldsymbol{\phi}_k\|_2^2 + \sum_{i\in\mathcal{S}\setminus\{k\}} |\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_i|^2 p_i & \ell = k \\ -|\boldsymbol{b}_k^{\mathrm{H}}\boldsymbol{W}_\ell\boldsymbol{\phi}_\ell|^2 p_k & \ell \neq k \in \mathcal{S} \\ 0 & \text{else,} \end{cases}$$

the matrix $\boldsymbol{\Phi}$ is column-wise diagonally dominant with positive diagonal entries and non-positive off-diagonal elements. Therefore, $\boldsymbol{\Phi}^{-1}$ exists and has non-negative entries. The resulting solution $\boldsymbol{a} = \boldsymbol{\Phi}^{-1}\boldsymbol{p}$ is non-negative, i.e., all α_k^2's are non-negative. In other words, any SINRs achievable in the dual MAC are also achievable in the original BC. From the multiplication of (14) with the all-ones vector, i.e.,

$$\mathbf{1}^{\mathrm{T}}\boldsymbol{\Phi}\boldsymbol{a} = [\|\boldsymbol{\phi}_1\|_2^2, \ldots, \|\boldsymbol{\phi}_K\|_2^2]\boldsymbol{a} = \mathbf{1}^{\mathrm{T}}\boldsymbol{p} = \sum\nolimits_{k=1}^K p_k,$$

we infer that, based on (13), $\mathrm{SINR}_k^{\mathrm{BC}} = \mathrm{SINR}_k^{\mathrm{MAC}}$ for all $k \in \{1, \ldots, K\}$ is always possible using the same transmit power $\sum_{k=1}^K \|\boldsymbol{\tau}_k\|_2^2$ in the BC as in the dual MAC.

Starting from given $\{\boldsymbol{\tau}_k\}_{k=1}^K$ in the BC and using $\boldsymbol{\phi}_k = \boldsymbol{\tau}_k\ \forall k$, it can be shown with similar steps as above that $\mathrm{SINR}_k^{\mathrm{MAC}} = \mathrm{SINR}_k^{\mathrm{BC}}$ can be accomplished for all $k \in \{1, \ldots, K\}$ by an appropriate choice for the MAC powers p_k. The resulting power allocations $\{p_k\}_{k=1}^K$ always exist and are non-negative. Moreover, $\sum_{k=1}^K p_k = \sum_{k=1}^K \|\boldsymbol{\tau}_k\|_2^2$ holds. This proves also the converse of the following theorem.

Theorem 2. *Any SINRs* (2) *and* (9) *are achievable in the vector BC with partial ZF using some total transmit power, iff the same SINRs* (12) *are achievable in the dual vector MAC employing the same total transmit power (cf. [8]).*

Due to Theorem 2, we will formulate and solve the problems in the dual vector MAC in the following. The BC solution can then be obtained with (13) and (14).

VII. Interference Function for Partial ZF

Whereas an SINR based formulation is essentially a precoder design in the BC, in the dual MAC it reformulates to a joint optimization of the transmit powers $\boldsymbol{p} = [p_1, \ldots, p_K]^{\mathrm{T}}$ and the adaptive receive strategies $\{\boldsymbol{\phi}_k\}_{k=1}^K$. For this joint optimization in the dual MAC, a generic framework with general interference functions of [5], [6], [14] can be applied. To this end, let us define the effective interference as

$$\mathcal{Z}_k(\boldsymbol{p}, \boldsymbol{\phi}_k) = \frac{\|\boldsymbol{\phi}_k\|_2^2 + \sum_{i\in\mathcal{S}\setminus\{k\}} |\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_i|^2 p_i}{|\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_k|^2},$$

such that $\mathrm{SINR}_k^{\mathrm{MAC}} = p_k/\mathcal{Z}_k(\boldsymbol{p}, \boldsymbol{\phi}_k)$ [cf. (12)]. As can be easily shown, the interference function

$$\boldsymbol{\mathcal{Z}}(\boldsymbol{p}, \boldsymbol{F}) = [\mathcal{Z}_1(\boldsymbol{p}, \boldsymbol{\phi}_1), \dots, \mathcal{Z}_K(\boldsymbol{p}, \boldsymbol{\phi}_K)]^{\mathrm{T}} \tag{15}$$

is standard (see [5]) for fixed equalizers $\boldsymbol{F} = [\boldsymbol{\phi}_1, \dots, \boldsymbol{\phi}_K]$ (see [6]), i.e., it satisfies the following three axioms:

A1. Positivity:	$\mathcal{Z}(\boldsymbol{p}, \boldsymbol{F}) > \mathbf{0}$	for $\boldsymbol{p} \geq \mathbf{0}$
A2. Monotonicity:	$\mathcal{Z}(\boldsymbol{p}, \boldsymbol{F}) \geq \mathcal{Z}(\boldsymbol{p}', \boldsymbol{F})$	for $\boldsymbol{p} \geq \boldsymbol{p}'$
A3. Scalability:	$\mu\mathcal{Z}(\boldsymbol{p}, \boldsymbol{F}) > \mathcal{Z}(\mu\boldsymbol{p}, \boldsymbol{F})$	for $\mu > 1$.

As has been demonstrated in [5], choosing the optimum equalizer also leads to a standard interference function, i.e.,

$$\mathcal{I}(\boldsymbol{p}) = [\min_{\boldsymbol{\phi}_1} \mathcal{Z}_1(\boldsymbol{p}, \boldsymbol{\phi}_1), \dots, \min_{\boldsymbol{\phi}_K} \mathcal{Z}_K(\boldsymbol{p}, \boldsymbol{\phi}_K)]^{\mathrm{T}} \tag{16}$$

is standard. This observation led to the algorithmic solutions in [6]. Note that the optimizer $\boldsymbol{\phi}_k$ for the kth element in (16) can be written as in Line 3 of Algorithm 1 and that $\mathcal{Z}_k(\boldsymbol{p}, \boldsymbol{\phi}_k)$ is independent w.r.t. a multiplication of the optimizer $\boldsymbol{\phi}_k$ with an arbitrary scalar $\mu_k \in \mathbb{C} \setminus \{0\}$.

VIII. Algorithmic Solution

Based on the duality of Section VI, the QoS optimization (8) can be equivalently formulated in the dual MAC:

$$\min_{\boldsymbol{p} \geq \mathbf{0}} \mathbf{1}^{\mathrm{T}}\boldsymbol{p} \qquad \mathrm{s.\,t.:} \quad \boldsymbol{p} \geq \mathrm{diag}(\boldsymbol{\gamma})\mathcal{I}(\boldsymbol{p}) \tag{17}$$

with the interference function $\mathcal{I}(\boldsymbol{p})$ introduced in (16) and the vector of SINR targets $\boldsymbol{\gamma} = [\gamma_1(\rho_1), \dots, \gamma_K(\rho_K)]^{\mathrm{T}}$, where $\gamma_k(\rho_k)$ is defined in (10). Due to the monotonicity property of $\mathcal{I}(\boldsymbol{p})$ together with the scalability property (cf. Axioms A2 and A3, respectively), the vector constraint in (17) is always active. This motivates the naturally arising fixed point iteration $\boldsymbol{p}^{(n)} = \mathrm{diag}(\boldsymbol{\gamma})\mathcal{I}(\boldsymbol{p}^{(n-1)})$, which was proven in [5] to converge to the global optimizer of (17).

Alternatively, we can employ the iteration proposed in [6] instead due to its quadratic convergence speed [14]. To this end, we reformulate (15) in matrix-vector notation, i.e.,

$$\mathcal{Z}(\boldsymbol{p}, \boldsymbol{F}) = \boldsymbol{\Psi}(\boldsymbol{F})\boldsymbol{p} + \boldsymbol{\xi}(\boldsymbol{F})$$

with $\boldsymbol{\xi}(\boldsymbol{F}) = [\xi_1, \dots, \xi_K]^{\mathrm{T}}$, $\xi_k = \|\boldsymbol{\phi}_k\|_2^2/|\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_k|^2$, and

$$[\boldsymbol{\Psi}(\boldsymbol{F})]_{k,\ell} = \begin{cases} \frac{|\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_\ell|^2}{|\boldsymbol{\phi}_k^{\mathrm{H}}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{b}_k|^2} & k \neq \ell \in \mathcal{S} \\ 0 & \text{else.} \end{cases}$$

In the optimum of (17), we have that

$$\mathrm{diag}(\boldsymbol{\gamma})\big(\boldsymbol{\Psi}(\boldsymbol{F})\boldsymbol{p} + \boldsymbol{\xi}(\boldsymbol{F})\big) = \boldsymbol{p}.$$

Rearranging this equation, leads to the fixed point equation in Line 4 of Algorithm 1. In Line 3, the optimal equalizers for the current power allocation are computed that minimize $\mathcal{Z}_k(\boldsymbol{p}^{(n-1)}, \boldsymbol{\phi}_k)$. As shown in [6], the iteration in Algorithm 1 converges to the global optimum of (17) when starting from an initial feasible point $\boldsymbol{p}^{(0)} \geq \mathrm{diag}(\boldsymbol{\gamma})\big(\boldsymbol{\Psi}(\boldsymbol{F}^{(0)})\boldsymbol{p}^{(0)} + \boldsymbol{\xi}(\boldsymbol{F}^{(0)})\big)$. Once Algorithm 1 has converged, the desired optimizers of (8) can be computed via the duality of Section VI.

Algorithm 1 Power Minimization

Require: SINR targets $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_K]^{\mathrm{T}}$, initial powers $\boldsymbol{p}^{(0)} \geq \mathrm{diag}(\boldsymbol{\gamma})\big(\boldsymbol{\Psi}(\boldsymbol{F}^{(0)})\boldsymbol{p}^{(0)} + \boldsymbol{\xi}(\boldsymbol{F}^{(0)})\big)$, accuracy ϵ

1: $n \leftarrow 1$ — initialize iteration counter
2: **repeat**
3: $\quad \forall k \in \{1, \dots, K\}$: — equalizer update
$\quad \boldsymbol{\phi}_k^{(n)} \leftarrow \big(\mathbf{I} + \sum_{i \in \mathcal{S}} \boldsymbol{W}_k \boldsymbol{b}_i \boldsymbol{b}_i^{\mathrm{H}} \boldsymbol{W}_k^{\mathrm{H}} p_i^{(n-1)}\big)^{-1} \boldsymbol{W}_k \boldsymbol{b}_k$
4: $\quad \boldsymbol{p}^{(n)} \leftarrow \big(\mathbf{I} - \mathrm{diag}(\boldsymbol{\gamma})\,\boldsymbol{\Psi}(\boldsymbol{F}^{(n)})\big)^{-1} \mathrm{diag}(\boldsymbol{\gamma})\,\boldsymbol{\xi}\big(\boldsymbol{F}^{(n)}\big)$ — fix point equation
5: $\quad n \leftarrow n + 1$ — increase iteration counter
6: **until** $\mathbf{1}^{\mathrm{T}}(\boldsymbol{p}^{(n)} - \boldsymbol{p}^{(n-1)})/\mathbf{1}^{\mathrm{T}}\boldsymbol{p}^{(n)} \leq \epsilon$
7: **return** $\boldsymbol{p}^{(n)}$

To find such an initial feasible power allocation, the authors of [6] suggest to use an SINR balancing strategy which solves a problem formulation that is dual to (17), namely,

$$\min_{\rho, \boldsymbol{p} \geq \mathbf{0}} \rho \qquad \mathrm{s.\,t.:} \quad \mathbf{1}^{\mathrm{T}}\boldsymbol{p} \leq P_{\mathrm{tx}}, \quad \rho\boldsymbol{p} \geq \mathrm{diag}(\boldsymbol{\gamma})\mathcal{I}(\boldsymbol{p}). \tag{18}$$

The optimal $\boldsymbol{p}$, that balances the SINR targets, satisfies

$$\rho \begin{bmatrix} \boldsymbol{p} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathrm{diag}(\boldsymbol{\gamma})\boldsymbol{\varXi}(\boldsymbol{F}) \\ \frac{1}{P_{\mathrm{tx}}}\mathbf{1}^{\mathrm{T}}\,\mathrm{diag}(\boldsymbol{\gamma})\boldsymbol{\varXi}(\boldsymbol{F}) \end{bmatrix}}_{\boldsymbol{\Theta}(\boldsymbol{F})} \begin{bmatrix} \boldsymbol{p} \\ 1 \end{bmatrix},$$

where $\boldsymbol{\varXi}(\boldsymbol{F}) = [\boldsymbol{\Psi}(\boldsymbol{F}), \boldsymbol{\xi}(\boldsymbol{F})]$. The first K rows of this eigenvalue equation follow from the vector constraint and the last row results from the power constraint in (18), that are both active in the optimum. Here, ρ and $[\boldsymbol{p}^{\mathrm{T}}, 1]^{\mathrm{T}}$ denote the spectral radius and the properly normalized dominant eigenvector of $\boldsymbol{\Theta}(\boldsymbol{F})$, respectively. This optimality condition motivates an alternating update procedure (cf. [6]):

1) For given $\boldsymbol{p}^{(n-1)}$, calculate $\{\boldsymbol{\phi}_k^{(n)}\}_{k=1}^K$ as in Line 3 of Algorithm 1.
2) Find the spectral radius $\rho^{(n)}$ and the properly normalized principle right eigenvector $[\boldsymbol{p}^{(n),\mathrm{T}}, 1]^{\mathrm{T}}$ of $\boldsymbol{\Theta}(\boldsymbol{F}^{(n)})$.

As soon as this procedure reaches a $\rho^{(n)} \leq 1$, we have found a power allocation $\boldsymbol{p}^{(n)} \geq \mathrm{diag}(\boldsymbol{\gamma})\big(\boldsymbol{\Psi}(\boldsymbol{F}^{(n)})\boldsymbol{p}^{(n)} + \boldsymbol{\xi}(\boldsymbol{F}^{(n)})\big)$ which may serve as an initialization for Algorithm 1. Otherwise, when the optimal value of (18) is $\rho > 1$, P_{tx} is smaller than the optimum of (17). A transmit power larger than the optimum of (17) may be found via successively increasing P_{tx} in (18) and repeatedly applying the SINR balancing strategy.

Applying the duality result of Section VI to the balancing formulation (11) leads to

$$\max_{\beta, \boldsymbol{p} \geq \mathbf{0}} \beta \quad \mathrm{s.\,t.:} \quad \mathbf{1}^{\mathrm{T}}\boldsymbol{p} \leq P_{\mathrm{tx}}, \quad \boldsymbol{p} \geq \mathrm{diag}(\boldsymbol{\gamma}(\beta))\mathcal{I}(\boldsymbol{p}) \tag{19}$$

with $\boldsymbol{\gamma}(\beta) = [\gamma_1(\beta\rho_1), \dots, \gamma_K(\beta\rho_K)]^{\mathrm{T}}$. Note that the rates are balanced in (19). Hence, the algorithmic solution for SINR balancing is not applicable.

As has already been mentioned at the end of Section IV, (19) can be solved with the help of (17). A bisection is performed in Algorithm 2 until the power minimization (17) has the given transmit power P_{tx} as its optimum. In other

Algorithm 2 Rate Balancing

Require: bounds β_{lower} and β_{upper}, SINR target function $\boldsymbol{\gamma} : \mathbb{R}_+ \to \mathbb{R}_+^K$ with $\beta \mapsto \boldsymbol{\gamma}(\beta)$, accuracy ϵ

repeat
  $\beta \leftarrow (\beta_{\text{upper}} - \beta_{\text{lower}})/2$
  $\rho, \boldsymbol{p} \leftarrow \text{SINR}(\boldsymbol{\gamma}(\beta), P_{\text{tx}})$ spectral radius optimization
  if $\rho > 1$ **or** $\mathbf{1}^{\text{T}}\boldsymbol{p} > P_{\text{tx}}$ **then**
    $\beta_{\text{upper}} \leftarrow \beta$ β is infeasible
  else
    $\beta_{\text{lower}} \leftarrow \beta$ β is feasible
  end if
until $\beta_{\text{upper}} - \beta_{\text{lower}} \leq \epsilon$
$\rho, \boldsymbol{p} \leftarrow \text{SINR}(\boldsymbol{\gamma}(\beta_{\text{lower}}), P_{\text{tx}})$
return β_{lower}, $\boldsymbol{p}$

words, the methods for the QoS power minimization problem can be used to test whether the targets $\boldsymbol{\gamma}(\beta)$ can be achieved with a total transmit power smaller than P_{tx} or not. In the remainder, this feasibility test is denoted as $\text{SINR}(\boldsymbol{\gamma}(\beta), P_{\text{tx}})$.

Here, we realized $\text{SINR}(\boldsymbol{\gamma}(\beta), P_{\text{tx}})$ via aforementioned spectral radius optimization of [6]. For the test, it is sufficient to check in each iteration whether $\rho^{(n)}$ is smaller or larger than one. If $\rho^{(n)} \leq 1$, then the current target vector $\boldsymbol{\gamma}(\beta)$ is feasible, otherwise, if the procedure converges to a $\rho > 1$, the targets are infeasible. Due to the knowledge of P_{tx} and $\boldsymbol{\gamma}(\beta)$, feasibility of the current β can be detected within a few iterations when initializing the procedure with $\boldsymbol{p} = \frac{P_{\text{tx}}}{K}\mathbf{1}$.

For the bisection in Algorithm 2, we also need an initialization of the lower and upper bound for β_{opt}. The lower bound β_{lower} must be feasible and is found with the rates of the uniform power allocation $\frac{P_{\text{tx}}}{K}\mathbf{1}$, whereas the upper bound β_{upper} must be infeasible and is found with the single-user rates. In the end of Algorithm 2, the optimal power allocation is computed based on β_{lower} that is feasible contrary to β_{upper}.

IX. Numerical Results

For a numerical verification of the proposed partial ZF method, we consider the following setup. A GEO-stationary satellite is directed to Munich (11°east and 48°north) and has a rectangular antenna array of $N = 100$ elements. The K perfect and statistical CSI users are randomly placed within the area from 35°east till 13°west and from 17°north to 70°north.

Within this geometric model, we used the free space path loss model for determining the values of $\{\boldsymbol{h}_k\}_{k\in\mathcal{S}}$ and $\{\boldsymbol{v}_k\}_{k\in\mathcal{M}}$. We considered a *fully loaded* scenario with $K = 100$ users, $|\mathcal{S}| = |\mathcal{M}| = 50$, and an *overloaded* scenario with $K = 120$ users, $|\mathcal{S}| = |\mathcal{M}| = 60$. In both cases, the users have targets $\rho_{2i-1} = 1$ and $\rho_{2i} = 2$ for $i \in \{1, \ldots, K/2\}$. For the figure, we generated 10 channel realizations and plotted the average balancing level β_{opt} versus P_{tx}/σ^2 in dB, $\sigma_k = \sigma\ \forall k$, for the following results: the results of (11), a total ZF strategy, and a tight upper bound on the maximum in (5) based on bounding the true $\bar{R}_k$ in (3) as $\bar{R}_k \leq \log_2\left(\frac{\psi_k^2|\boldsymbol{v}_k^{\text{H}}\boldsymbol{t}_k|^2}{\sigma_k^2 + \sum_{i\neq k}\psi_k^2|\boldsymbol{v}_k^{\text{H}}\boldsymbol{t}_i|^2}\right)$.

Figure 1 shows the results for the fully loaded setup (solid curves), where β_{opt} grows unboundedly with P_{tx}/σ^2, and the overloaded setup (dashed curves), where β_{opt} saturates for high P_{tx}/σ^2. We see that partial ZF clearly outperforms the total ZF strategy that is close to zero even in the medium P_{tx}/σ^2 regime due to the low angular spread of the users from the satellites perspective. Note that ZF w.r.t. all users, except the intended one, is impossible in the overloaded setup.

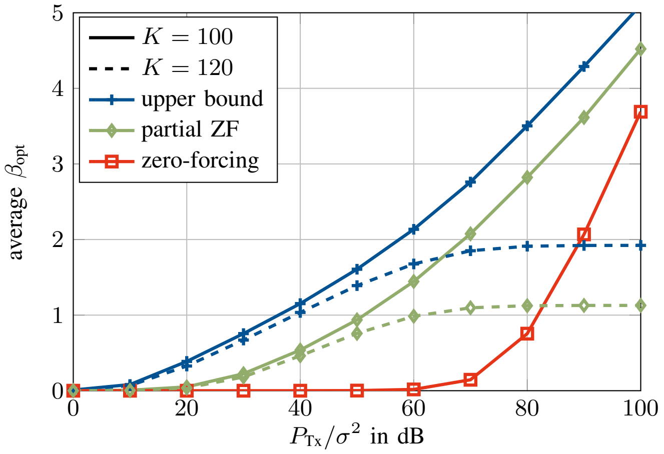

Fig. 1. Average balancing level β_{opt} in a fully loaded system, $K = 100$ users, and an overloaded system, $K = 120$ users, with $N = 100$ antennas.

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